

$$\sin^5 x + \cos^5 x = 1$$

1 попытка

$$\left(\frac{1-\cos 2x}{2}\right)\left(\frac{3\sin x-\sin 3x}{4}\right)+\left(\frac{1+\cos 2x}{2}\right)\left(\frac{3\cos x+\cos 3x}{4}\right)=1$$

$$3\sin x-3\cos 2x\sin x-1\sin 3x+1\cos 2x\sin 3x+3\cos x+3\cos 2x\cos x+\cos 3x+\cos 2x\cos 3x=8$$

2 способ

$$\sin^5 x + \cos^5 x = \sin^2 x + \cos^2 x$$

$$\sin^5 x - \sin^2 x = \cos^2 x - \cos^5 x$$

$$\sin^2 x(\sin^3 x - 1) = \cos^2 x(1 - \cos^3 x)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\sin^2 x((\sin x - 1)(\sin^2 x + \sin x + 1)) = \cos^2 x(1 - \cos x)(\cos^2 x + \cos x + 1)$$

$$(1 - \cos^2 x)((\sin x - 1)(\sin^2 x + \sin x + 1)) = (1 - \sin^2 x)(1 - \cos x)(\cos^2 x + \cos x + 1)$$

$$((1 - \cos x)(1 + \cos x))((\sin x - 1)(\sin^2 x + \sin x + 1)) = ((1 - \sin x)(1 + \sin x))(1 - \cos x)(\cos^2 x + \cos x + 1)$$

$$((1 - \cos x)(1 + \cos x))((\sin x - 1)(\sin^2 x + \sin x + 1)) + ((\sin x - 1)(1 + \sin x))(1 - \cos x)(\cos^2 x + \cos x + 1) = 0$$

$$((1 - \cos x)(\sin x - 1)) \quad ((1 + \cos x)(\sin^2 x + \sin x + 1) + (1 + \sin x)(\cos^2 x + \cos x + 1)) = 0$$

$$(1 - \cos x)(\sin x - 1) = 0$$

$$1 - \cos x = 0$$

$$\cos x = 1$$

$$x = 2\pi k$$

$$\sin x = 1$$

$$x = \pi/2 + 2\pi k$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1) + (1 + \sin x)(\cos^2 x + \cos x + 1) = 0$$

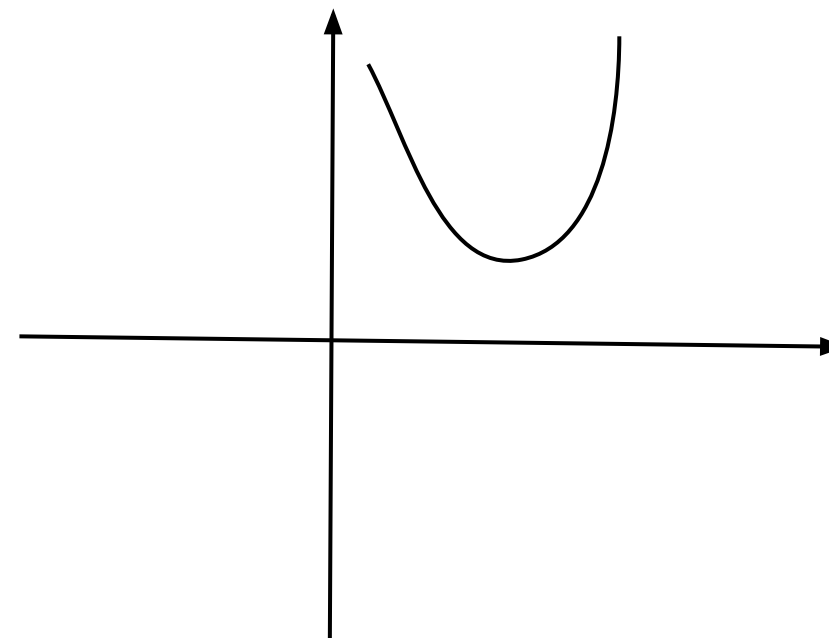
$$f(x) = \sin^2 x + \sin x + 1 \quad f(x) > 0 \quad g(x) = \cos^2 x + \cos x + 1 > 0$$

$$\sin x = t$$

$$f(t) = t^2 + t + 1$$

$$D = 1 - 4 = -3$$

Ответ: $2\pi k$; $\pi/2 + 2\pi k$;



$$1 + \cos x > 0$$

$$\cos x > -1$$

$$1 + \sin x < 0$$

$\sin x < -1$ - не бывает

